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The high-frequency finite-temperature quark dispersion relation

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Abstract

I calculate the dispersion relation for quarks of mass m and momentum k in a quark gluon plasma at temperature T, in the limit $m^2 + k^2 \gg (gT)^2$, where g is the strong coupling constant. I find three contributions to the dispersion relation: one that depends on T but not m or k, one that depends on m and T but not k, and third contribution that depends on all three (and is opposite in sign to the other two).

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Recently, there has been much interest in calculating the production rates for both massless and massive quarks in ultrarelativistic nuclear collisions [1-6]. Unfortunately, there have been no rigorous thermal field theory calculations to compare to approximate results, probably because the "naive" production rates diverge for massless quarks. The production rates can be regularized by summing over hard thermal loops [7, 8], following the treatment of Braaten and Pisarski [9]. However, this is a difficult calculation, and has not been done so far.

The closest approach to a full calculation has been to treat the thermal quark masses for massless quarks as if they were bare masses, in order to regulate the divergent production rates [1]. However, the thermal corrections for massive quarks are not included, and these can be important when $m \sim T$. There have been several papers in which the thermal dispersion relation was calculated for massive quarks [10-15], but these have concentrated on the low-frequency dispersion relation. Also, no calculations have been done for systems out of chemical equilibrium, while in the early stages of an ultrarelativistic nuclear collision the quarks (and anti-quarks) have densities below their chemical equilibrium values.

Because the main interest here is calculating effective masses to regulate production rates, I calculate the dispersion relation in the high-frequency limit. This is because the divergences in the rates occur at small values of t or u, but at arbitrary values of s, so the quarks involved have momenta of order $(T^2 + mT)^{1/2}$ rather than of order gT. I calculate the thermal masses for arbitrary bare mass, following the calculation of Petitgirard [12] and working to order g^2, to provide a smooth transition from the production of massless quarks to that of massive quarks.

I begin with a quark gluon plasma of temperature T with strong coupling constant g. To one-loop order, the quark self-energy $\Sigma(K)$ can be written as

$$\Sigma(K) = -a\,\not{K} - b\,\not{u} - cm, \tag{1}$$

where

$$\begin{aligned} a &= \frac{1}{4k^2}\left[\mathrm{Tr}\,(\not{K}\mathrm{Re}\Sigma) - \omega\mathrm{Tr}\,(\not{u}\mathrm{Re}\Sigma)\right], & (2)\\ b &= \frac{1}{4k^2}\left[(\omega^2 - k^2)\mathrm{Tr}\,(\not{u}\mathrm{Re}\Sigma) - \omega\mathrm{Tr}\,(\not{K}\mathrm{Re}\Sigma)\right], & (3)\\ c &= \frac{1}{4m}\mathrm{tr}\,(\mathrm{Re}\Sigma)\,. & (4) \end{aligned}$$

Here the quark four-momentum $K = (\omega, \vec{k})$, u is the matter four-velocity and m is the bare quark mass. The Lorentz-invariant denominator of the quark propagator is then

$$\mathrm{Tr}\,(\not{K} - m - \Sigma)^2 = (1+a)^2K^2 + 2(1+a)bK_\mu u^\mu + b^2 - (1-c)^2m^2, \tag{5}$$

and its poles give the quark dispersion relation.

The dispersion relation in the plasma rest frame is thus [12],

$$(1+a)\omega + b = \sqrt{(1+a)^2k^2 + (1+c)^2m^2}. \tag{6}$$

Because $\Sigma = \mathcal{O}(g^2)$, I rewrite the dispersion relation (6) as

$$\omega = \sqrt{k^2 + m^2 + 2(c-a)m^2 - \frac{2b}{\omega}(k^2+m^2)} + \mathcal{O}(g^3), \tag{7}$$

for $\omega > k \gg gT$. I then use the relation

$$\omega^2 = k^2 + m^2 + \mathcal{O}(g^2) \tag{8}$$

to obtain

$$\omega = \sqrt{k^2 + m^2 + \delta} + \mathcal{O}(g^3), \tag{9}$$

where

$$\delta = 2(c-a)m^2 - 2b\omega = \frac{m}{2}\mathrm{tr}\,(\mathrm{Re}\Sigma) + \frac{1}{2}\mathrm{Tr}\,(\not{K}\mathrm{Re}\Sigma)\,. \tag{10}$$

The self-energy projections have been calculated by Petitgirard [12],

$$\begin{aligned}
\mathrm{Tr}\,(\not{K}\mathrm{Re}\Sigma) &= 2g^2C_F\int_0^\infty \frac{pdp}{(2\pi)^2}\left[\left(4 + \frac{K^2+m^2}{2pk}L_B(p)\right)n_B(p)\right.\\
&\quad \left. + \left(4p + \frac{K^2+m^2}{2k}L_F(p)\right)\frac{n_F(E)}{E}\right], &(11)\\
\mathrm{tr}\,(\mathrm{Re}\Sigma) &= \frac{4g^2C_F}{k}\int_0^\infty \frac{pdp}{(2\pi)^2}\left[-L_B(p)\frac{n_B(p)}{p} + L_F(p)\frac{n_F(E)}{E}\right], &(12)
\end{aligned}$$

where C_F is the quadratic Casimir invariant of the quark representation, $n_{B(F)}$ is the Bose (Fermi) distribution function, $E = (p^2+m^2)^{1/2}$,

$$\begin{aligned}
L_B(p) &= \ln\left|\frac{[K^2 - m^2 + 2p(\omega+k)]\,[K^2-m^2-2p(\omega-k)]}{[K^2-m^2+2p(\omega-k)]\,[K^2-m^2-2p(\omega+k)]}\right|, &(13)\\
L_F(p) &= \ln\left|\frac{[K^2+m^2+2(E\omega+pk)]\,[K^2+m^2-2(E\omega-pk)]}{[K^2+m^2+2(E\omega-pk)]\,[K^2+m^2-2(E\omega+pk)]}\right|. &(14)
\end{aligned}$$

As the logarithms are already multiplied by g^2, and I am interested only in the lowest order corrections to the dispersion relation, I take $K^2 = m^2$ in the logarithms, which is equivalent to using the identity (8), obtaining the simpler forms

$$\begin{aligned}
L_B(p) &= 0, &(15)\\
L_F(p) &= 2\ln\left|\frac{p-k}{p+k}\right|. &(16)
\end{aligned}$$

The thermal correction to the bare mass is then

$$\begin{aligned}
\delta &= g^2C_F\int_0^\infty \frac{dp\,p}{(2\pi)^2}\left[4n_B(p) + \left(4 + \frac{2m^2}{pk}\ln\left|\frac{p-k}{p+k}\right|\right)\frac{pn_F(E)}{E}\right], &(17)\\
&= g^2C_F\left[\left(\frac{1}{6} + q(\frac{m}{T})\right)T^2 + r(\frac{m}{T},\frac{k}{T})m^2\right], &(18)\\
q(\frac{m}{T}) &= \frac{1}{\pi^2T^2}\int_m^\infty dE\,p\,n_F(E), &(19)\\
r(\frac{m}{T},\frac{k}{T}) &= \frac{1}{2\pi^2\,k}\int_m^\infty dE\,\ln\left|\frac{p-k}{p+k}\right|\,n_F(E). &(20)
\end{aligned}$$

It is clear from inspection that $q > 0$ and $r < 0$.

For regulation of production rates in the early stages of ultrarelativistic nuclear collisions, the most interesting case is $\mu < 0$. The case $\mu > 0$ is discussed thoroughly in Refs. [11] and [15]. For simplicity, I calculate the average value of r,

$$\overline{r}(\frac{m}{T}) = \frac{\int \frac{d^3k}{\omega}\, r(\frac{m}{T}, \frac{k}{T})\, n_F(\omega)}{\int \frac{d^3k}{\omega}\, n_F(\omega)}, \tag{21}$$

where $\omega = (K^2 + m^2)^{1/2}$. Using $\overline{r}$ for regulation of divergences is probably the most sensible procedure, in the absence of a full calculation with effective thermal propagators and vertices.

The most useful case for present simulations is when the gluons are chemically equilibrated but the quark densities are very low ($-\mu/T \gg 1$), as these are the conditions under which most quark production occurs in an ultra-relativistic nuclear collision. In this case,

$$\overline{\delta}(\frac{m}{T}) = g^2 C_F \left[\frac{T^2}{6} + e^{\mu/T}\left(T^2 q_\infty(\frac{m}{T}) + m^2 \overline{r}_\infty(\frac{m}{T})\right)\right], \tag{22}$$

$$q_\infty(\frac{m}{T}) = \frac{1}{\pi^2 T^2}\int_m^\infty dE\, p\, e^{-E/T}, \tag{23}$$

$$\overline{r}_\infty(\frac{m}{T}) = \frac{\int_m^\infty d\omega \int_m^\infty dE\, \ln\left|\frac{p-k}{p+k}\right|\, e^{-(E+\omega)/T}}{2\pi^2 \int_m^\infty d\omega\, k\, e^{-\omega/T}}. \tag{24}$$

The integrals can be evaluated in closed form in the low- and high-mass limits:

$$q_\infty\left(\frac{m}{T}\right) = \begin{cases} \frac{1}{\pi^2}\left(1 + \frac{m^2}{2T^2}\ln(\frac{m}{T}) + \mathcal{O}[\frac{m^2}{T^2}]\right) & m/T \ll 1, \\ \left(\frac{m}{2\pi^3 T}\right)^{1/2} e^{-m/T}\left(1 + \frac{3T}{8m} + \mathcal{O}[\frac{T^2}{m^2}]\right) & m/T \gg 1; \end{cases} \tag{25}$$

$$\overline{r}_\infty\left(\frac{m}{T}\right) = \begin{cases} \frac{-1}{2\pi^2}\left(1 - \frac{m^2}{T^2}\ln^2(\frac{m}{T}) + \mathcal{O}[\frac{m^2}{T^2}\ln(\frac{m}{T})]\right) & m/T \ll 1, \\ -\left(\frac{T}{8\pi^3 m}\right)^{1/2} e^{-m/T}\left(1 - \frac{19T}{8m} + \mathcal{O}[\frac{T^2}{m^2}]\right) & m/T \gg 1. \end{cases} \tag{26}$$

They can also be easily evaluated numerically to high accuracy. I was unable to find accurate interpolation formulas, probably because of the non-analytic behavior of q_∞ and r_∞ at $m = 0$.

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